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FREE, PLANE, LAMINAR JET OF AN INCOMPRESSIBLE FLUID IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD

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FREE, PLANE, LAMINAR JET OF AN INCOMPRESSIBLE FLUID IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD

René Moreau

ABSTRACT

Certain results achieved by the related solution established in a previous paper (Ref. 1) are analyzed and the fundamental hypothesis of the boundary layer is discussed.

The particular distributions of a transverse magnetic field of the form $B = kx^p$ is considered here, for which the velocity along the axis of the jet is (Ref. 1):

if $p \neq -2/3$:

$$u_0 = cx^{-\frac{1}{3}}\left[1 - \frac{\sigma k^2}{\rho c}\frac{x^{2p+\frac{4}{3}}}{2p+\frac{4}{3}}\right]; \tag{1}$$

if $p = -2/3$:

$$u_0 = cx^{-\frac{1}{3}}\left[1 - \frac{\sigma k^2}{\rho c}\operatorname{Log}_e x\right]. \tag{2}$$

As a function of the above, the principal laws of flow are expressed in simple fashion:

velocity profile:

$$l = \sqrt{\frac{6\nu x}{u_0}}; \tag{3}$$

effective width:

$$\zeta = \frac{y}{l(x)}; \tag{4}$$

[redacted]

entrained quantity:

$$Q = \int_0^\infty u\,dy = \sqrt{6\nu u_0 x}; \tag{5}$$

momentum:

$$\mathcal{J} = \int_0^\infty \rho u^2\,dy = \frac{2}{3}\rho u_0 \sqrt{6\nu u_0 x}; \tag{6}$$

current lines:

$$\zeta = \operatorname{arg\,th} \frac{\psi}{\sqrt{6\nu u_0 x}}. \tag{7}$$

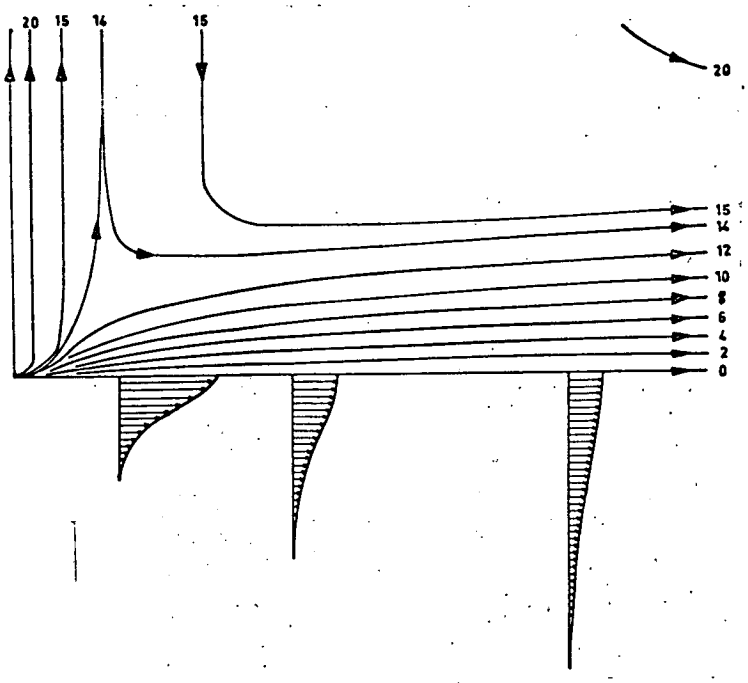
It is clear that the flow is not defined except in the regions where $u_0 x \geqslant 0$. This condition is always satisfied in nonconductor fluids in which $u_0 = cx^{-1/3}$, but here, for the values $p \geqslant -2/3$, a cutoff section appears in which the jet is completely stopped and the current lines are deflected in the direction of the applied magnetic field. Four types of flow can be distinguished according to the law of applied magnetic field or according to the value of p.

Type a: $p < -1$. The electromagnetic action, which is very strong at the start, decreases more rapidly than the action of the viscosity. An infinitely large quantity must be injected, of which a part is also turned in the direction of the applied field, whereas the central core may entrain the external fluid in the direction of the jet when the viscosity becomes preponderant. In the above example, $p = -3/2$. /2

Type b: $p = -1$. A finite quantity is injected, around which the viscosity may entrain external fluid from the origin onward. The electromagnetic action causes a rapid widening of the jet near the slot. /3

Type c: $-1 < p < -2/3$. A flux of infinite momentum is injected through the slot, but the exterior fluid entrained by the viscosity is the only source of flow. In the above example, $p = -5/6$.

Type d: $p \geqslant -2/3$. Near the origin, the action of the viscosity is preponderant at the same time as the flow has the usual appearance of a jet in nonconductor fluids. The electromagnetic action, however, increases more rapidly, broadens the jet progressively, and goes as far as to deflect it completely toward the lines of force of the magnetic field. The quantity entrained in the direction of the jet passes thus through a maximum and is reduced to zero in the section of stoppage. In the above example, $p = 0$.



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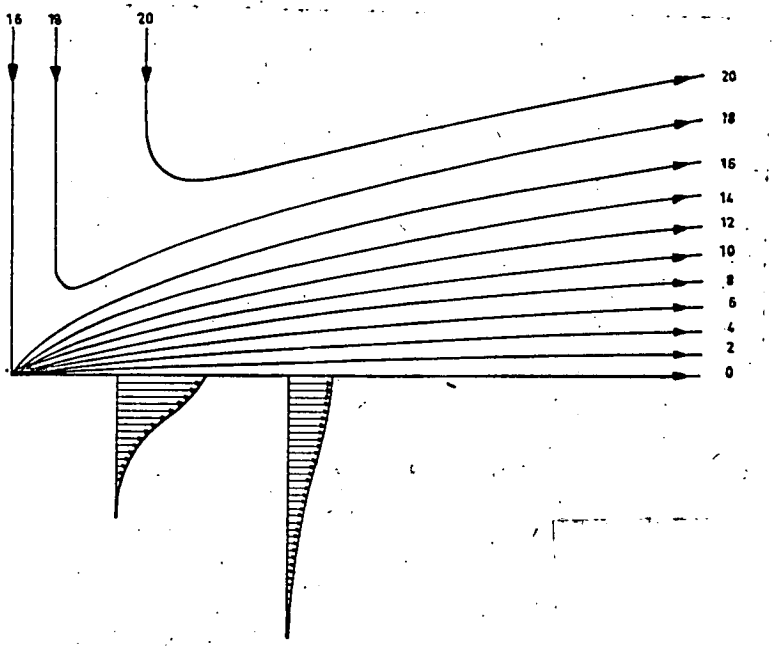

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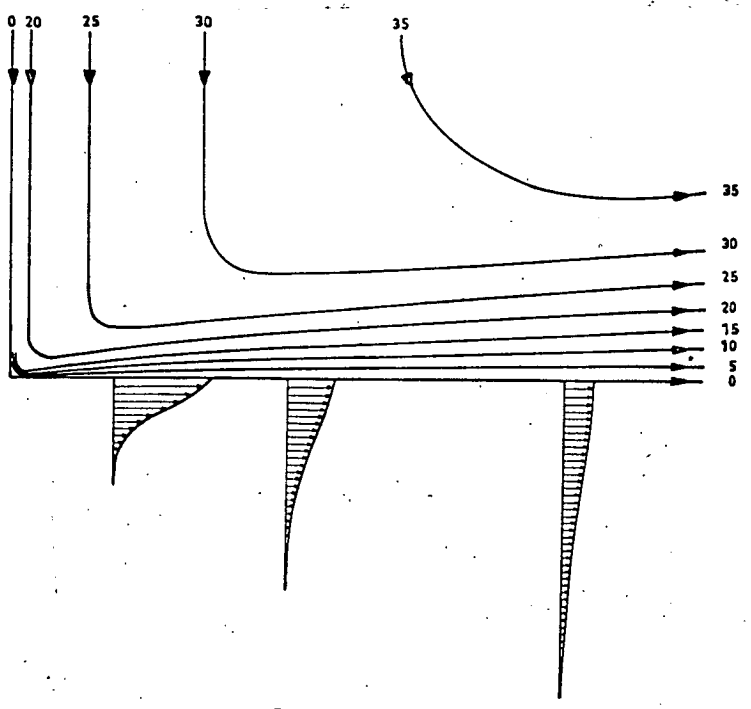

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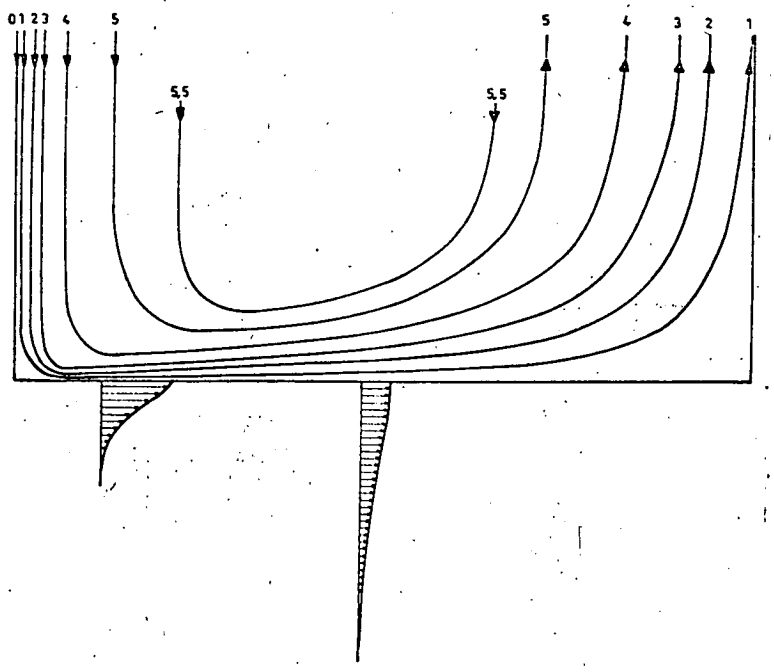

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The solution given by Jungclaus (Ref. 2) for $p < -2/3$ corresponds, within the more general family just examined, to the particular value $c = o$. This implies that all the parameters of flow are bound essentially to the magnetic field and become zero at the same time as it does. Our solution with $c \neq o$ brings us back always to that of the free jet of nonconductor fluids (Ref. 3) if the magnetic field is reduced to zero.

It should be noted that the approximation of the boundary layer, the fundamental hypothesis in this investigation, is lacking both in the section of origin--whether the fluid is a conductor or not--and in the section of stoppage when $p \geqslant -2/3$ exists. This is of no importance at the origin, because this solution could not represent a true jet except at a certain distance beyond its potential cone. The same cannot be said concerning the area of the section of stoppage. It can be thought, indeed, that by means of the term $\nu(\partial^2 u/\partial x^2)$, which has been disregarded in this investigation, the viscosity might prevent the total stoppage of the jet at a finite distance. One limit of the validity of our solution may be evaluated by discovering in what section the following relation is no longer negligible. /4

$$\frac{\left(\frac{\partial^2 u}{\partial x^2}\right)_{y=0}}{\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0}} = -\frac{3\nu u_0'' x}{u_0^2}. \qquad (8)$$

Thus, in the case of a uniform magnetic field, it may be seen that when this limit is reached, the momentum flux is only a very small fraction (about 5 percent) of what is injected, whereas the quantity in the direction of the jet is still appreciable (about 30 percent of the maximum quantity). The velocity profile is then very wide and very flat.

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References

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2. Jungclaus, G. Rev. Mod. Phys., Vol. 32, 823-827, 1960.
3. Schlichting, H. Boundary Layer Theory, 143-146, 1955.

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